higher momenta, although the statistical accuracy of the measurements for $t > 0.5$ $(\text{BeV}/c)^2$ is rather poor. Three points from the CERN experiment[3] at 4.0 BeV/c are shown in Fig. 1 in support of this conclusion. The over-all behavior of the cross section as a function of energy and t seems to be very similar to that observed in the πN experiments. The cross section for $p\bar{p}$ scattering at 1.75 BeV/c from this experiment is also shown in Fig. 1 for comparison. All the data of our experiment are compatible with measurements made at other laboratories for this cross section, and no indication of a minimum appears in any of these data. These results may, of course, be compatible with diffraction theory[7,8]; however, if Regge theory is working at these low energies, as is indicated in other experiments, then this experiment, as explained in the following Letter,[4] offers additional evidence in its support.

A more complete set of data will be published at a later date.

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REGGE TRAJECTORIES AND MINIMA IN DIFFERENTIAL CROSS SECTIONS*

Steven Frautschi†
California Institute of Technology, Pasadena, California
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Attempts to fit[1] the earliest accurate measurements of high-energy cross sections, namely the total cross sections and diffraction peaks, by Regge poles have not been totally convincing because a number of high-spin trajectories contribute, and the theory involves a correspondingly large number of parameters. Some of the smaller cross sections now being measured, however, can be described in terms of only one or two trajectories, thus providing a cleaner test of Regge theory.[2-4] For example, only the ρ among known trajectories contributes to $\pi^- + p \to \pi^0 + n$,[2] and the experiments on this reaction have confirmed the striking, qualitative features of a single-Regge-pole model: the energy dependence at $t = 0$[2] and the shrinking forward peak.[5] In addition, the helicity-flip amplitude vanishes when the exchanged ρ trajectory passes through spin zero, giving rise to a minimum in the differential cross section at $t \approx -0.6$ $(\text{BeV})^2$.[5-7] The purpose of this Letter is to emphasize that minima in $d\sigma/dt$ near $t = -0.6$ $(\text{BeV})^2$ associated with passage of exchanged spins through zero may be very common, that study of this easily recognized qualitative feature promises to become a major aid in unravelling the details of Regge trajectories even in cases where several trajectories contribute, and that in particular, the minimum of $d\sigma(p + \bar{p} \to p + \bar{p})/dt$ reported in the accompanying Letter[8] may have this origin.

To illustrate the Regge description of the minima near $t = -0.6$ $(\text{BeV})^2$, we shall first review the situation for $\pi^- + p \to \pi^0 + n$ and then proceed to the progressively more complicated reactions $\pi^\pm + p \to \pi^\pm + p$ and $p + \bar{p} \to p + \bar{p}$.

$\pi^- + p \to \pi^0 + n$.—The ρ is the only known trajectory contributing to this reaction at small

t. Comparing the one-pole relation $d\sigma(s)/dt = f(t)\exp\{[2\alpha_\rho(t)-2]\ln s\}$ with the measurements at 3 to 18 BeV/c,[2,5] one finds that $\alpha_\rho(t)$ has decreased from 1 at $t=m_\rho^2\approx 0.6$ (BeV)^2 to about $\frac{1}{2}$ at $t=0$ and continues to decrease steadily, passing through zero near $t=-0.6$ (BeV)^2. At the zero of α_ρ, the helicity-flip amplitude is predicted to vanish.[6,7] Now from the tendency of $d\sigma/dt$ to rise[2,5] at very small t before the exponential falloff sets in, one may infer[1,6,7] that the helicity-flip amplitude is much larger than -nonflip; this conclusion is supported by study of the ρ contributions to nucleon electromagnetic form factors[9] and by analysis of backward np scattering.[10] Thus the zero of the helicity flip leads to a definite minimum of $d\sigma/dt$, which is indeed observed at $t\approx -0.6$ (BeV)^2 for all energies [Fig. 1(a)].

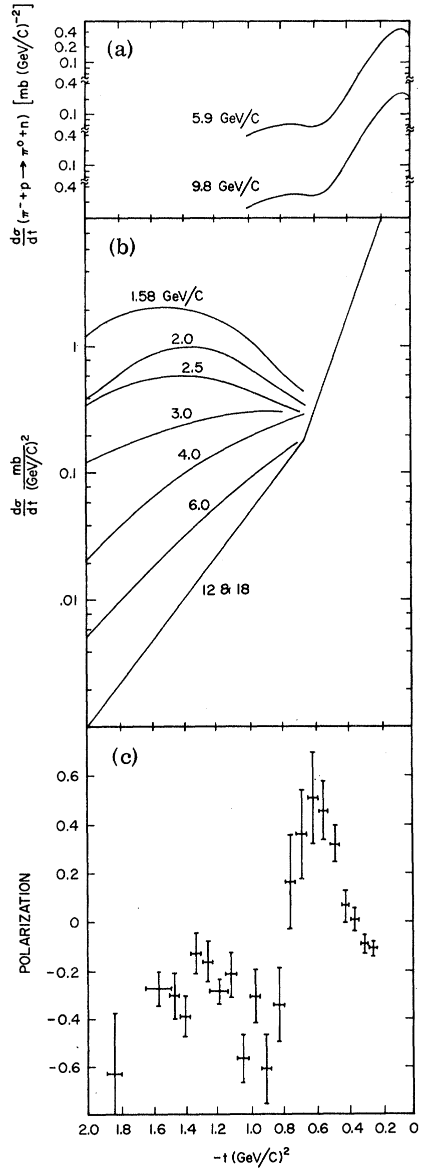

FIG. 1. (a) $d\sigma(\pi^- + p \rightarrow \pi^0 + n)/dt$ at 5.9 and 9.8 BeV/c (Ref. 5). (b) $d\sigma(\pi^- + p \rightarrow \pi^- + p)/dt$ at 1.6-18.0 BeV/c (Ref. 20). (c) Polarization in $\pi^- + p \rightarrow \pi^- + p$ at 2.1 BeV/c (Ref 16).

Now the analysis of other reactions usually involves more trajectories. From Fig. 2, however, one sees that of the known meson trajectories with $\alpha(t=0)>0$ (P; 2^+ nonet with $S=0$ members $T_1(P'), T_8, A_2$; 1^- nonet with $S=0$ members V_1, V_8, ρ),[11] most or possibly all except P pass through $\alpha=0$ in the general vicinity of $t=-0.5$ (BeV)^2. Moreover, the helicity-flip amplitude associated with exchange of each of the vector nonet (V) trajectories vanishes at $\alpha=0$ It is an open theoretical question whether exchange of the 2^+ nonet (T) trajectories can produce helicity flip at $\alpha=0$[12]; for the purposes of the discussion, we shall suppose they cannot. Thus, dips of $d\sigma/dt$ in the vicinity of $t=-0.5$ (BeV)^2 are likely to occur whenever helicity flip is important, even though several trajectories may be interfering.[13,14] From the occurrence (or absence) and relative sizes of the secondary peaks following the dips in different reactions, one can test the scheme presented in Fig. 2 and infer the relative strengths (or destructive interference) of helicity flip associated with different trajectories. One can also check experimentally whether helicity flip due to T exchange indeed vanishes at $\alpha=0$, which will be of great interest for the theory of ghost-killing.[12] Let us see how this works

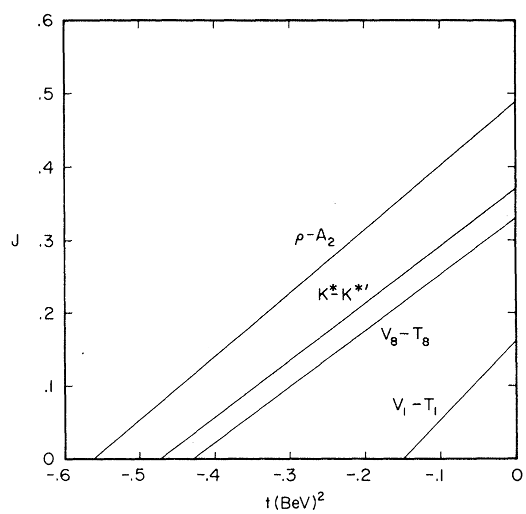

FIG. 2. A tentative plot of leading meson trajectories, obtained by extrapolating the straight-line trajectories of Ref. 11.

out in πN, pp, and $p\bar{p}$ scattering.

$\pi^{\pm}+p\rightarrow\pi^{\pm}+p$.—Here, any of the $G=+$ trajectories, P, P', T_8, and ρ, can be exchanged. At 2 BeV/c,[15] a large polarization is observed,[16] which requires a large helicity flip that is out of phase with a comparably large nonflip term.[17] Presumably the main nonflip term is P exchange; the out of phase helicity flip then has to be associated with secondary trajectories. Since the secondary trajectories all pass through zero, the helicity-flip amplitude is expected to change sign somewhere near $t=-0.5$ (BeV)^2. Experimentally, both $d\sigma(\pi^{+}+p\rightarrow\pi^{+}+p)/dt$ [18,19] and $d\sigma(\pi^{-}+p\rightarrow\pi^{-}+p)/dt$ [20] do have minima around $t\approx-0.6$ (BeV)^2, followed by similar-appearing secondary maxima, at energies up to about 3 BeV/c [Fig. 1(b)]. The helicity-flip zero is further confirmed by the t dependence of the polarization at 2 BeV/c, which changes sign near the minimum of $d\sigma/dt$ [Fig. 1(c)].[16] The secondary peak melts steadily as the energy rises and reduces to a shoulder above about 3 BeV/c; this is consistent with the notion that this peak is caused by secondary trajectories whose influence relative to P exchange decreases as the energy rises. The polarization also decreases considerably as the energy rises.[21,22]

The trajectory mainly responsible for the helicity flip could be either ρ ($C=-$) or one of the secondary 2^+ trajectories ($C=+$). To see this, note that the phase of a Regge term at $t\leq0$ is given by the signature factor, which is

$$(1-e^{-i\pi\alpha}) \tag{1}$$

for ρ and other members of the vector nonet (V), and

$$(1+e^{-i\pi\alpha}) \tag{2}$$

for P and the 2^+ nonet (T). If all secondary trajectories have similar $\alpha(t)\approx\alpha_P(t)-\frac{1}{2}$, then both ρ and the 2^+ secondary trajectories are out of phase with P (by 45°, or 135°, $\cdots$) in a way suitable for polarization to occur. Furthermore, V and T are 90° out of phase with each other, so the near equality[18-20] of the secondary peak heights in $\pi^{+}+p\rightarrow\pi^{+}+p$ and $\pi^{-}+p\rightarrow\pi^{-}+p$ does not rule out their simultaneous presence even though V and T have opposite C. A T trajectory probably does dominate, though, since the strength of ρ-exchange helicity flip as deduced from the height of the secondary peak in πN charge exchange is several times too small to explain the secondary peak or shoulder in elastic πN scattering at the same energy.[23] This conclusion can be checked when the $\pi^{+}p$ polarization is measured: If a $C=+$ ($-$) trajectory dominates, the polarization should have the same (opposite) sign for $\pi^{+}p$ and elastic $\pi^{-}p$.

$p+p\rightarrow p+p$ and $p+\bar{p}\rightarrow p+\bar{p}$.—All the leading meson trajectories contribute to these reactions. In the imaginary part of the forward nonflip amplitude, as deduced via the optical theorem from σ(total), the secondary trajectories of opposite C cancel for pp and add for $p\bar{p}$.[24] In the helicity-flip amplitude, the large helicity flip in the πN secondary peak suggests the possibility of a similarly large term in NN and $N\bar{N}$. Experimentally, $d\sigma(p\bar{p})/dt$ has a minimum at $t=-0.5$ (BeV)^2 for $p_{\text{lab}}=1.5$ to 2.75 (BeV/c), as reported in the accompanying Letter[8]; this is so similar to $d\sigma(\pi N)/dt$ that one immediately suspects helicity-flip phenomena. There is no secondary peak in $d\sigma(pp)/dt$,[25] on the other hand, and the pp polarization though substantial at $-t=0.3$ (BeV)^2 is small at $-t\geq0.6$ (BeV)^2 over the range $p_{\text{lab}}=2$ to 6 (BeV/c).[26] These facts suggest that for helicity flip at $-t\geq0.5$ (BeV)^2 (at least, for the part which is 90° out of phase with P exchange), the secondary trajectories of opposite C again tend to cancel for pp and add for $p\bar{p}$.

If this interpretation of the dip in $d\sigma(p\bar{p})/dt$ is correct, one expects that: (i) The dip–secondary-peak sequence, being associated with secondary trajectories, should go away with increasing energy as it did in elastic πp scattering. It need not go away at exactly the same rate since different interferences may be involved. (ii) Where the secondary peak is prominent, $p\bar{p}$ scattering should exhibit a sizeable polarization as did πp scattering.

Detailed analysis of the helicity flip presents challenges. As mentioned earlier, if the main secondary trajectories share a common $\alpha(t)$, then the $C=-$ exchanges are 90° out of phase with the $C=+$ exchanges and cannot cancel them in the pp helicity-flip amplitude [the parts which are 90° out of phase with P exchange can cancel, however, thus explaining the small pp polarization; similarly the cancellation in the optical theorem works because it involves only the imaginary parts of (1) and (2)]. It may well be that interference[27] with some other exchange such as P is important in helicity flip. For a full understanding of the situation, pp and $p\bar{p}$ elastic scattering will have to be supplemented by studies of polarization, combined with pn charge exchange and $p+\bar{p}\to n+\bar{n}$, which depend only on the $I=1$ exchanges.

In addition to the cases discussed above, many other reactions are presumably affected by the passage of the secondary trajectories through spin zero. For example, in the ABBBHLM collaboration,[28] one finds some indication of dips at $p_{\text{lab}}=4$ BeV/c, $t\approx-0.5$ (BeV)^2 in $\pi^++p\to p^++p$, $\pi^++p\to\pi^0+N^{*++}$, and $\pi^++p\to\rho^0+N^{*++}$. On the other hand, the dip expected from A_2 exchange is not seen in $\pi^-+p\to\eta+n$.[3] More detailed experiments on all these reactions should prove most illuminating in sorting out the secondary trajectories and their helicity-flip couplings.

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[12]When members of the 2^+ nonet pass through spin 0, the signature factor [Eq. (2) of text] does not give a zero cancelling the Regge pole as it did for the vector nonet [Eq. (1) of text]. Then since the passage through spin 0 occurs at negative t, the famous ghost-killing zero is needed to cancel the pole. If the amplitude kills the ghost by simply "choosing nonsense" [M. Gell-Mann, in Proceedings of the International Conference on High-Energy Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 539], the helicity-flip amplitude (which is "nonsense" at $\alpha=0$) comes out finite but nonvanishing at $\alpha=0$. But if the amplitude "choose sense," and then an additional zero occurs in both helicity flip and nonflip at $\alpha=0$ [as implied by the ghost-killing mechanism of G. Chew, Phys. Rev. Letters 16, 60 (1966)], the helicity-flip term can actually vanish at $\alpha=0$.

[13]Exceptions can occur when the helicity flip is due to P or π exchange.

[14]In addition, there are further zeroes in amplitudes such as $\pi+A\to\omega+B$; see L. Wang, Phys. Rev. Letters 16, 756 (1966).

[15]We shall continue the Regge analysis right down to 2 BeV/c here. There are several cases [A. Carroll et al., Phys. Rev. Letters 16, 288 (1966); V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966)] where this seems to work well, provided one keeps the highest spin resonances in the direct channel in addition to the leading Regge exchanges. The fluctuations in energy due to the direct channel resonances have no great effect on $d\sigma/dt$ except at backward angles [S. Kormanyos et al., Phys. Rev. Letters 16, 709 (1966)], and we can ignore the resonances in the discussion of $t\approx-0.6$ (BeV)^2.

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[17]Note that the single-ρ-exchange model for $\pi^-+p\to\pi^0+n$ predicts no polarization because the flip and nonflip terms for a given trajectory have the same phase.

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[22]We will not go into the further details of the polarization, which seem rather complicated.

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